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Passive Nutation Damping Using Large Appendages with Application to Galileo

Douglas E. Bernard*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

In the analysis of many passive nutation dampers, spacecraft dynamics and damper dynamics may be considered decoupled although some coupling will always be present. When large appendages are used for nutation damping, this coupling must be included in the analysis. In this paper, passive nutation damping using large appendages is analyzed for oblate spinning spacecraft such as Galileo—the Jupiter Orbiter/Probe. This analysis yields approximate analytical expressions for dynamic response to torques and accelerations; stability parameters such as wobble amplification, and the time constant for nutation decay. The time constant is determined using a modification of the energy sink method to include the coupled dynamics. Large flexible appendages are seen to be a potential source of dynamic instability. Accordingly, the nutation damper design for spacecraft with such appendages becomes a tradeoff between nutation damper time constant and dynamic stability margin. The Galileo design is described including tradeoffs and other practical considerations.

Nomenclature

A,B,C = spacecraft inertia about x, y, and z axes, respectively, when the appendage is undeflected ($\beta=0$)
A_w = wobble amplification matrix
b = nutation damper dashpot constant in N-m/(rad/s) (the dashpot torque will be $-b\dot{\beta}$)
b_0 = "optimal" b
E = system mechanical energy = $T+U$
E_a = mechanical energy available for nutation damping
g = $-(J/A)[A+(B-C)(J'/J)]\Omega$
H = angular momentum
i = index which refers to the ith body
 = 1 for S/C minus appendage
 = 2 for appendage
J_0 = combined effect of all sources of flexibility other than the nutation damper on the (1,1) term of A_w
J = $I_{2zz}-I_{2yy}+mR_{2y}(-R_{1y}+R_{2y})$
J^* = $I_{2xx}+mR_{2y}^2$
J' = $I_{2xx}+mR_{2y}(-R_{1y}+R_{2y})$
K = nutation damper spring constant in N-m/rad (the spring torque will be $-K\beta$)
L = $T-U$
m_i = mass of body i
m = reduced mass = $m_1m_2/(m_1+m_2)$
q_j = generalized coordinate j
Q_j = generalized force corresponding to generalized coordinate j
R_i = vector from hinge point to body i
T = system kinetic energy of rotation
T_z,T_x,T_y,T_β = contributors to Q
T_{ST} = stiction torque
U = system potential energy (internal springs)
u = $K+J\Omega^2-(JJ'/A)\Omega^2-\lambda^2(J^*-J'^2/A)$
$\dot{V}_Z$ = axial acceleration of spacecraft
v = $b\lambda$
X,Y,Z = spacecraft coordinate axes
β = appendage deflection angle with respect to the spacecraft
β_w,β_n = wobble and nutation portions of β, respectively
Γ = defined in Eq. (26)-$f(A,B,C,J,J',J^*,\Omega,b,K)$
ζ = stability parameter ($\zeta<1$ for stability)
θ = nutation angle
θ_{xR} = residual nutation due to stiction
λ = precession frequency observed from rotor frame
λ_0 $\equiv\sqrt{\dfrac{(C-A)(C-B)}{BA}}\Omega$
τ = nutation damping time constant
τ_0 = time constant for $b=b_0$
ϕ,θ_1,θ_2 = "312" Euler angle set describes body 1 orientation with respect to inertial space
ω_1 = angular velocity vector of body 1 $\equiv[\omega_x,\omega_y,\omega_z]$
ω_w,ω_n = wobble and nutation portions of ω_1, respectively
ω_T = instantaneous envelope of ω_y
Ω = constant average value of ω_z (spin frequency)

I. Introduction

ONE task common to all spin-stabilized spacecraft is the active or passive control of nutation. A common passive nutation damping scheme is the use of compact, damped pendulums. Both the Galileo and the NASA International Solar Polar Mission (ISPM) spacecraft achieve nutation damping based on the damped pendulum concept; but each uses a large existing appendage as the pendulum rather than a dedicated mass. This paper presents the analysis and design of such a nutation damping system. The development is general enough to be applied to a large class of spacecraft, and has been used in the design of the dual-spin Galileo spacecraft and the analysis of the ISPM spacecraft.

Previous work[1] published on damped-pendulum nutation dampers has been for "simple pendulum" nutation dampers, where the pendulum is compact relative to spacecraft dimensions. A large appendage is likely to be a compound (extended) pendulum, which couples dynamically with the nutational motion of the spacecraft. A sufficiently large appendage, free to move with respect to the remainder of the spacecraft, can jeopardize the dynamic stability of the spacecraft.

In order to design the nutation damper efficiently and effectively, predictions of the resulting spacecraft dynamics in analytical form are useful, if not vital.

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*Senior Engineer, Guidance and Control Section. Member AIAA.

Accordingly, while retaining the use of computer simulations for final design verification and fine tuning, the analysis described below was performed to allow simple (though approximate) closed-form expressions for nutation damping time constant, optimal damping constant, body-fixed torque response, and translational acceleration response. In addition, expressions are developed for stability and wobble amplification and residual nutation, given stiction.

The results presented here allow the straight-forward design specification for passive nutation dampers with large appendages, and are used in the design of the Galileo nutation damper.

This paper is organized as follows.

A closed-form expression for the nutation damping time constant is developed in Sec. II. While not exact, this expression has been found to have sufficient accuracy over a broad range of spacecraft configurations, and to be extremely useful in the design process. The development resembles an energy sink analysis, with the following exception: the energy sink assumption that nutation damper and spacecraft dynamics are decoupled is not made. For large appendages, this assumption is known not to be true. An alternative simplifying assumption, which allows the predominant effects of the coupling to be felt, is that the solution to one of the equations of motion is known and simple (sinusoidal). The frequency of the sinusoid is determined, including flexible body effects.

Section III demonstrates that there will exist an optimum damping constant resulting in a minimum time constant for a given spacecraft configuration and nutation damper spring constant. A procedure is outlined for determining that damping constant value. A very simple first approximation is provided.

The effect of the nutation damper on stability is investigated in Sec. IV. The choice of the nutation damper spring constant is shown to be a direct tradeoff between time constant and stability margin when using large appendages.

Section V describes and analyzes a number of other practical matters which must be considered during the nutation damper design. Among these are stiction, trap states (for dual spinners), and dynamic response to external torques and forces.

The design of a nutation damping system using the results presented here is described in Sec. VI, with particular attention paid to necessary tradeoffs and design drivers. The Galileo nutation damper design is used as an example.

II. Time Constant

In developing an expression for time constant, the following criteria were used: *Accuracy*—The expression must predict the time constant with "sufficient" accuracy for use in design and analysis. *Analytical Expressibility*—If possible, the time constant should be computable in a non-recursive manner, using elementary functions. Its partial derivatives should be similarly expresible. *Simplicity*—The expression should be as simple as possible for ease of use, and to allow an intuitive understanding of time constant dependencies. *Generality*—The expression should be applicable to a wide range of single- and dual-spin spacecraft.

The class of spacecraft for which this expression may be used is loosely described as follows: 1) They are stable in the presence of energy dissipation. 2) They each include one large appendage used for nutation damping. This appendage, with one principal axis nominally perpendicular to the S/C spin axis, possesses one hinge degree-of-freedom with respect to the spacecraft. The hinge is perpendicular to both the spacecraft spin axis and the appendage principal axis described above. 3) Any hinge torque is defined by a linear spring, a linear dashpot, and stiction. (The stiction will be ignored except for calculations of the residual nutation.)

The derivation will proceed as follows:

A) Generation of the equation of motion for the four degree-of-freedom system (three rigid-body-rotational degrees-of-freedom; one hinge degree-of-freedom).

B) Approximate solution of the equations of motion.

C) Calculation of average energy dissipation rate over one nutation cycle as a function of the parameter ω_T.

D) Calculation of mechanical energy available for dissipation as a function of the parameter ω_T.

E) Formation of the expression for nutation damping time constant—seen to be independent of the parameter ω_T.

A. Equations of Motion

In the following, Dj is used to indicate definition j, and Rj to indicate restriction j. Consider two arbitrary bodies (1 and 2) connected by a single degree-of-freedom hinge of arbitrary location and orientation. Body 2 will be the large appendage and body 1 the remainder of the spacecraft.

D1: Define the body 2 Y-axis (Y_2) as the body 2 axis of minimum moment of inertia.

R1: The hinge line must be perpendicular to Y_2 (the body 2 axis of minimum moment of inertia).

D2: Define the body 1 and body 2 X-axes (X_1 and X_2) parallel to the hinge line. The body 2 Z-axis may be formed as: $Z_2 = X_2 \times Y_2$.

D3: Define the body 1 Z-axis, Z_1 such that when Z_1 is parallel to Z_2, the axis of maximum moment of inertia of the composite spacecraft is parallel to Z_1 and Z_2. Form $Y_1 = Z_1 \times X_1$.

D4: Define β as the angle from Y_1 to Y_2. β will be positive when $Y_2 \cdot Z_1 > 0$.

D5: Define the hinge point on the hinge line such that $R_{1x} = 0$, where R_{1x} is the X-component of the vector R_1 from the hinge point to the body 1 c.m. in body 1 coordinates.

D6: Define R_2 as the vector from the hinge point to the body 2 c.m. in body 2 coordinates.

R2: $R_{2x} = 0$, (D5 + D6 + R2) $\Rightarrow$ the plane defined by $X = 0$ contains the hinge point and the body 1 and body 2 c.m. for all β.

R3: $R_{2z} = 0$. (This restriction makes $\beta = 0$ a stable equilibrium point for spin about Z.)

R4: $I_{2xz} = 0$, where I_2 is the inertia of body 2 about its own c.m. in body 2 coordinates.

Definitions 1 and 3 and restriction 4 allow us to write the body 2 and spacecraft inertia matrices when $\beta = 0$ with some terms known to be zero:

$$I_2 = \begin{bmatrix} I_{2xx} & 0 & 0 \\ 0 & I_{2yy} & 0 \\ 0 & 0 & I_{2zz} \end{bmatrix} \qquad I = \begin{bmatrix} A & I_{xy} & 0 \\ I_{xy} & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

Similarly, from definition 5 and restrictions 2 and 3:

$$R_1 = [0, R_{1y}, R_{1z}]^T \qquad R_2 = [0, R_{2y}, 0]^T$$

Figure 1 is a schematic of the system under consideration. Both rotary spring and dashpot are assumed to act at the hinge with spring constant K and damping constant b.

D7: Define V_1 and V_2 as the velocities of the c.m. of bodies 1 and 2, with respect to some inertial reference frame. Define V:

$$V = V_2 - V_1$$

D8: Define ω_1 as the angular velocity of body 1; ω_2 as the angular velocity of body 2.

From the above definitions:

$$V = \dot{r} + \omega_1 \times r$$

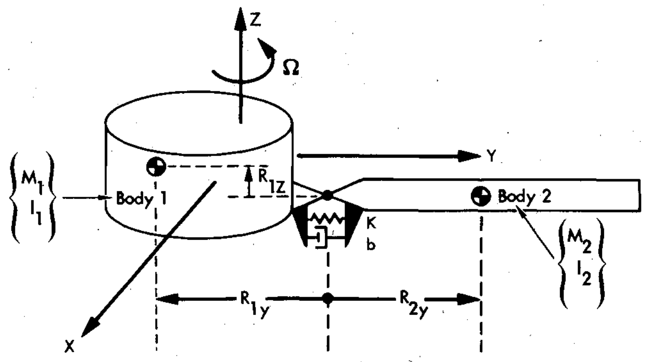

Fig. 1 Spacecraft schematic.

where r is the distance to the c.m. of body 2 from the c.m. of body 1 in the body 1 reference frame.

R5: The nutation angle, θ, about Z is small:

$$\theta,\beta \ll 1 \qquad \omega_x,\omega_y,\dot{\beta} \ll \omega_z$$

Define the generalized coordinates as the set of "312" Euler angles defining the spacecraft orientation with respect to inertial space (see Nomenclature) augmented by the boom deflection angle, β.

$$q=[\phi,\theta_1,\theta_2,\beta]^T$$

The only nonconservative force acting continuously will be the dashpot force. Other nonconservative forces may act from time to time such as when thrusters are fired. The set of generalized nonconservative forces may be written

$$Q=[0,0,0,-b\dot{\beta}]^T+[T_z,T_x,T_y,T_\beta]^T$$

We may now proceed with a Lagrangian formulation of the equations of motion:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\left(\frac{\partial L}{\partial q_j}\right)=Q_j \qquad (j=1,2,3,4) \tag{1}$$

Two additional restrictions are required to allow the expression of Eq. (1) in a sufficiently simple form:

R6: $T_z=0$ (no spin up/down torques)

R7: $I_{xy}\ll(C-A)$ $\qquad I_{xy}\ll(C-B)$

The first order approximation to Eq. (1) becomes

$$\omega_z=\Omega=\text{const} \tag{2}$$

$$A\dot{\omega}_x+J'\ddot{\beta}+(C-B)\Omega\omega_y+J\Omega^2\beta=T_x \tag{3}$$

$$B\dot{\omega}_y+(A-C)\Omega\omega_x+(J'-J)\Omega\dot{\beta}=T_y \tag{4}$$

$$J^*\ddot{\beta}+J'\dot{\omega}_x+J\Omega\omega_y+(K+J\Omega^2)\beta+b\dot{\beta}=T_\beta \tag{5}$$

The torques T_x, T_y, and T_β can cause spacecraft wobble, as well as nutation. If these torques are constants or step functions, the wobble will manifest itself as a DC component in ω_x,ω_y, and β. If this DC term is subtracted out, Eqs. (3-5) can be rewritten in terms of the nutational components only, which are the components of interest in this derivation. Define:

$$\omega_x=\omega_{xn}+\omega_{xw} \qquad \omega_y=\omega_{yn}+\omega_{yw} \qquad \beta=\beta_n+\beta_w$$

where

$$\omega_{xw}=-T_y/(C-A)\Omega$$

$$\omega_{yw}=\frac{T_x(K+J\Omega^2)/\Omega-T_\beta J\Omega}{(K+J\Omega^2)(C-B)-J^2\Omega^2}$$

$$\beta_w=\frac{-T_xJ+T_\beta(C-B)}{(K+J\Omega^2)(C-B)-J^2\Omega^2} \tag{6}$$

Substituting into Eqs. (3-5):

$$A\dot{\omega}_{xn}+J'\ddot{\beta}_n+(C-B)\Omega\omega_{yn}+J\Omega^2\beta_n=0 \tag{7}$$

$$B\dot{\omega}_{yn}+(A-C)\Omega\omega_{xn}+(J'-J)\Omega\dot{\beta}_n=0 \tag{8}$$

$$J^*\ddot{\beta}_n+J'\dot{\omega}_{xn}+J\Omega\omega_{yn}+(K+J\Omega^2)\beta_n+b\dot{\beta}_n=0 \tag{9}$$

When using these equations, we should define the generalized coordinates and forces as

$$q_n=[\phi,\theta_{1n},\theta_{2n},\beta_n] \qquad Q_n=[0,\ 0,\ 0,-b\dot{\beta}_n]$$

where θ_{1n}, θ_{2n} correspond to the body rates ω_{xn} and ω_{yn}.

B. Approximate Solution of Equations of Motion

The equations derived in Sec. II.A may be used to generate the time history of the system response to arbitrary initial conditions. They may also be used to generate stability criteria. One additional simplification is required, however, to allow a useful closed-form expression for nutation damping time constant.

Assumption: Over a single body fixed nutation cycle, ω_{yn} acts as a sinusoidal forcing function at the steady-state nutation frequency, λ:

$$\omega_{yn}(t)=\omega_T\cos\lambda t \tag{10}$$

When the time constant τ is evaluated, this assumption can be partially validated by showing

$$1-e^{(-2\pi/\lambda\tau)}\ll 1$$

Using Eq. (10) as a forcing function, Eqs. (7) and (9) may be decoupled, and we may write the solution for β_n:

$$\beta_n=\frac{g\omega_T(v\sin\lambda t+u\cos\lambda t)}{u^2+v^2} \tag{11}$$

where

$$g\equiv-(1/A)[JA+J'(B-C)]\Omega \tag{12}$$

$$u\equiv K+J\Omega^2-JJ'\Omega^2/A-(\lambda^2/A)(J^*A-J'^2) \tag{13}$$

$$v\equiv b\lambda \tag{14}$$

The steady-state nutation frequency of the flexible spacecraft may differ significantly from the rigid body nutation frequency if the appendage is large. The rigid body nutation frequency, λ_0, is found by writing Eqs. (7) and (8) for $\beta=\dot{\beta}=\ddot{\beta}=0$ in the following form:

$$\dot{\omega}_{xn}+\lambda_{x0}\omega_{yn}=0 \qquad \dot{\omega}_{yn}-\lambda_{y0}\omega_{xn}=0$$

and expressing λ_0:

$$\lambda_0=\sqrt{\lambda_{x0}\lambda_{y0}}=\sqrt{\left(\frac{C-B}{A}\right)\left(\frac{C-A}{B}\right)}\Omega$$

Taking $\lambda=\lambda_0$ is sufficiently accurate for estimation of time constants, except when very large appendages are used or when the stability margin is small. In such cases, the effect of the flexibly attached appendage on spacecraft dynamics must be considered when evaluating λ. A simple improved estimate may be formed by substituting Eqs. (10) and (11) and their derivatives into Eqs. (7) and (8), and assuming (consistent with the assumption of the form of ω_{yn}) that the system is very underdamped or $b \cong 0$.

Writing the resulting equations in the form

$$\dot{\omega}_{xn} + \lambda_x \omega_{yn} = 0 \qquad \dot{\omega}_{yn} - \lambda_y \omega_{xn} = 0$$

λ becomes

$$\lambda^2 = \lambda_x \lambda_y = \lambda_0^2 \left[1 + \frac{g(J\Omega^2 - J'\lambda^2)}{u(C-B)\Omega} \right] \left[1 + \frac{(J'-J)g\Omega}{uB} \right]^{-1}$$

Note that λ appears on the right-hand side of this equation, both explicitly and implicitly via u. While this equation may be solved analytically for λ, the resulting expression is too unwieldy for practical use. An alternate approach is a numerical solution which uses λ_0 as a first estimate. Define:

$$u_0 = K + J\Omega^2 - JJ'\Omega^2/A - (\lambda_0^2/A)(J^*A - J'^2)$$

Then, use the first iteration as the improved estimate.

C. Calculation of the Average Energy Dissipation Over One Nutation Cycle ($0 \le t \le 2\pi/\lambda$)

At any given time, the instantaneous rate at which energy is dissipated is the sum of the products of the nonconservative generalized forces Q_{nj} with the time derivatives of the generalized coordinates q_{nj}:

$$\dot{E} = \sum_j Q_{nj} \frac{\mathrm{d}q_{nj}}{\mathrm{d}t} = Q_{n4}\dot{q}_{n4} \tag{15}$$

From Sec. II.A, $q_{n4} = \beta_n$, $Q_{n4} = -b\dot{\beta}_n$, so

$$\dot{E} = -b\dot{\beta}_n^2 \tag{16}$$

This defines the instantaneous rate of energy dissipation. Over a single nutation cycle ($0 < t < 2\pi/\lambda$), the average rate of energy dissipation may be calculated by

$$\overline{\dot{E}} = \frac{\lambda}{2\pi} \int_0^{2\pi/\lambda} (-b\dot{\beta}^2)\,\mathrm{d}t = -\frac{b}{2} \frac{(g\lambda\omega_T)^2}{u^2+v^2} \tag{17}$$

D. Calculation of the Mechanical Energy Available for Dissipation

The mechanical energy:

$$E = T + U \tag{18}$$

will be formed at $t = t_0 = 2n\pi/\lambda$ $n \in \{0,1,2,...\}$ and at $t = t_f$; $t_f \to \infty$.

The angular momentum, H, will also be formed at t_0 and t_f, allowing the mechanical energy available for dissipation, E_a, to be computed by invoking conservation of angular momentum.

Under the same restrictions invoked in Sec. II.A, this may be expressed in the body 1 coordinate system as

$$H = \begin{bmatrix} A\omega_x + J'\dot{\beta} \\ B\omega_y - J\beta\omega_z \\ -J\beta\omega_y + (C - J\beta^2)\omega_z \end{bmatrix} \tag{19}$$

At $t = t_f$, all nutation has been damped out and

$$\omega_x = \omega_y = \dot{\beta} = \beta = 0 \tag{20}$$

and Eqs. (18) and (19) give

$$E_f = \tfrac{1}{2} C\omega_{zf}^2 \tag{21}$$

$$H_f^2 = C^2 \omega_{zf}^2 \tag{22}$$

Defining the available energy E_a as

$$E_a(t) = E(t) - E_f = E(t) - H^2/2C \tag{23}$$

$$\begin{aligned} 2CE_a(t) &= \omega_x^2(AC - A^2) + \omega_y^2(BC - B^2) \\ &+ \omega_z^2(\beta^2)(JC - J^2) + \dot{\beta}^2(J^*C - J'^2) \\ &+ 2\dot{\beta}\omega_x(CJ' - AJ') + 2\omega_y\omega_z(J\beta)B + \beta^2 KC \end{aligned} \tag{24}$$

Using Eq. (10) for ω_y; $\dot{\omega}_y$ is zero at $t = t_0$, and Eq. (8) becomes

$$\omega_x(t_0) = \left(\frac{J'-J}{C-A}\right)\dot{\beta}$$

Evaluating Eqs. (10) and (11) at $t = t_0$ yields

$$\omega_y(t_0) = \omega_T(t_0) \quad \beta(t_0) = \frac{gu\omega_T(t_0)}{u^2+v^2} \quad \dot{\beta}(t_0) \frac{g\lambda v\omega_T(t_0)}{u^2+v^2}$$

Using the above to evaluate Eq. (24) at $t = t_0$:

$$E_a(t_0) = \tfrac{1}{2}\omega_T^2(t_0)\Gamma \tag{25}$$

where

$$\begin{aligned} \Gamma = \Bigg\{ &\frac{B}{C}(C-B) + \frac{(K + J\Omega^2 - (J^2/C)\Omega^2)u^2g^2}{(u^2+v^2)^2} \\ &+ \frac{(J^*C - J'^2)v^2\lambda^2g^2}{C(u^2+v^2)^2} + \frac{2J\Omega(B/C)ug}{u^2+v^2} \\ &+ \frac{[(J'-J)^2A/(C-A) + 2J'(J'-J)]v^2\lambda^2g^2}{(u^2+v^2)^2} \Bigg\} \end{aligned} \tag{26}$$

Note for long, thin appendages, $J' \cong J$ and the last term in the above equation is zero.

E. Calculation of the Nutation Damping Time Constant

The nutation damping time constant, τ, is defined such that after one time constant, the nutation angle will be only $1/e$ times its initial value, assuming exponential decay.

For a single cycle:

$$\omega_T(t_0 + 2\pi/\lambda) = e^{-2\pi/\lambda\tau}\omega_T(t_0) \tag{27}$$

Thus over that cycle:

$$\overline{\dot{\omega}_T} = -(1/\tau)\omega_T \qquad \overline{\dot{E}_a} = -2E_a(t_0)/\tau \tag{28}$$

or

$$\tau = -2E_a/\overline{\dot{E}_a} \tag{29}$$

Substituting from Eqs. (17) and (25) gives a computable expression for the nutation damping time constant:

$$\tau = (2b\Gamma/g^2)[(u/v)^2 + 1] \tag{30}$$

III. Optimal Damping Constant

The optimum damping constant may be formally described as the value which b assumes when:

$$\frac{\partial\tau}{\partial b} = 0 \qquad K, \Omega, \text{inertia} = \text{const} \tag{31}$$

Unfortunately, Eq. (31) cannot be solved for b in a simple, closed form. If we simplify the expression for τ, we will be able to solve Eq. (31) for $b=b_0$ with b_0 considered a first approximation to the optimal b. Increased accuracy may then be obtained by iteration on the original Eq. (31) if desired.

The required simplification is that we assume that Γ is only a weak function of b. This is true for Galileo.

With constant Γ, the value of b at which Eq. (31) is satisfied is easily calculated to be

$$b_0 = u/\lambda \tag{32}$$

or

$$v_0 = u \tag{33}$$

yielding

$$\tau_0 = (4\Gamma/g^2)b_0 \tag{34}$$

Expressing the time constant nondimensionally using Eqs. (30) and (34):

$$\tau/\tau_0 = \tfrac{1}{2}(b_0/b + b/b_0) \tag{35}$$

IV. Stability and Wobble Amplification

Wobble amplification[2] relates the wobble observed in a flexible body to that found in the same body, assuming no flexibility (rigid body):

$$\begin{bmatrix} \theta_{1w} \\ \theta_{2w} \end{bmatrix} = A_w \begin{bmatrix} \theta_{1w0} \\ \theta_{2w0} \end{bmatrix} \tag{36}$$

where A_w is the wobble amplification matrix. For the spacecraft as defined and restricted in Sec. II, A_w is found to be

$$A_w = \begin{bmatrix} \dfrac{(C-B)(K+J\Omega^2)}{(C-B)(K+J\Omega^2)-J^2\Omega^2} & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \tag{37}$$

If other sources of flexibility besides the nutation damper exist, their effects on the (1,1) term of A_w can be collapsed into a single term, with the units of moment of inertia. Calling this term J_0, A_{w11} can be modified to read

$$A_{w11} = \frac{C-B}{C-B-J^2\Omega^2/(K+J\Omega^2)-J_0} \tag{38}$$

This leads to the stability criteria:

$$\zeta + J_0/(C-B) < 1.0 \tag{39}$$

where

$$\zeta \equiv \frac{J^2\Omega^2}{(K+J\Omega^2)(C-B)} \tag{40}$$

Note that for $J<C-B-J_0$ and positive K, Eq. (39) is always satisfied even as K approaches zero. For such J's no spring is required to assure stability. J is an inertia property related to appendage inertia, so for $[C-B-J_0]>0$, appendages below a certain "size" cannot cause instability, and need not include a spring-like restoring force. Conversely, the "larger" the appendage, J, the greater the spring constant, K, needed to assure stability.

That a tradeoff is needed between stability margin and time constant may be seen by rewriting Eq. (34)—the first approximation to the minimum time constant—as a function of ζ. The coefficients are not important, but the result has the form

$$\tau_0 = c_1(1/\zeta) + c_2 \tag{41}$$

where the coefficients are functions of inertia properties and spin rate only, and, therefore, less accessible as design parameters. τ_0 is minimized by maximizing ζ while, from Eqs. (38-40), A_{w11} is minimized by minimizing ζ.

V. Practical Considerations

Although the preceding sections form the basis from which the design of the passive nutation damper may proceed, there are a number of other practical issues which must be considered in the course of that design.

A. Stiction

The nutation damper performance for very small angle nutation will be limited by stiction. Below a given nutation angle, the spring- and spin-dynamical torques on the nutation damper will be insufficient to overcome stiction, and the relative motion between spacecraft and appendage will cease ($\dot{\beta}=\ddot{\beta}=0$). When the appendage comes to rest, the stiction torque is resisting a constant torque proportional to the final boom deflection angle β, and an approximately sinusoidal torque proportional to the nutation angle. The value of the residual nutation angle depends on the final boom deflection angle, but is largest when $\beta=0$ and the entire stiction torque is available to resist the torque proportional to the nutation angle.

For the case of $\beta=\dot{\beta}=\ddot{\beta}=0$, Eqs. (7) and (8) become Euler's equations in ω_{xn}, ω_{yn}. From Euler's equations, the instantaneous nutation angle when $\omega_x=0$ is

$$\theta_{xn} = B\omega_{yn}/C\Omega \tag{42}$$

The spin-dynamical torque on the appendage about its hinge for $\beta=0$ is, from Eq. (9)

$$T_\beta = J'\dot{\omega}_{xn} + J\omega_{yn}\Omega$$

Substituting for $\dot{\omega}_x$ from Euler's equations and using the definition of g:

$$T_\beta = g\omega_{yn} \tag{43}$$

Substituting Eq. (42) into Eq. (43) and setting the stiction torque, T_{ST}, equal to T_β at $\theta_{xn}=\theta_{xnR}$ gives

$$\theta_{xnR} = (B/C\Omega)(T_{ST}/g) \tag{44}$$

B. Trap States

Adding flexibility at the base of an appendage can increase the despin motor torque required to avoid minimum energy trap states for dual-spin vehicles. A minimum energy trap state can occur when a dual-spin vehicle is in all-spin configuration, and the despin motor has limited torque. The torque required to avoid a trap state depends on rotor and stator static imbalances, dynamic imbalances, and nonsymmetries.[3] Rotor appendage flexibility may drastically increase the dynamic imbalance of the rotor. The change in rotor product of inertia and therefore dynamic imbalance will be

$$\Delta I_{yz}^{\text{rotor}} = -J\beta \tag{45}$$

Avoidance of this trap state must be considered in the nutation damper design.

C. Body Fixed Torque (BFT) Response

In a rigid body, the dynamic response to a body fixed torque about the x- or y-axis has both nutation frequency (λ) and wobble frequency (0 rad/s) components when viewed in the spacecraft reference frame. The flexible body response may be described simply as follows: 1) The nutation frequency component is damped. 2) The wobble frequency component is amplified.

The general body fixed torque case considered consists of

$$Q=[0,\ T_x,\ T_y,\ -b\dot{\beta}]^T$$

Evaluating Eq. (6) for this case gives

$$\omega_{xw}=\frac{-T_y}{(C-A)\Omega}=\frac{-T_y}{(C-A)\Omega}A_{w22}$$

$$\omega_{yw}=\frac{(K+J\Omega^2)T_x}{\Omega[(K+J\Omega^2)(C-B)-J^2\Omega^2]}=\frac{T_x}{\Omega(C-B)}A_{wII}$$

$$\beta_w=\frac{-JT_x}{[(K+J\Omega^2)(C-B)-J^2\Omega^2]} \tag{46}$$

Note from Eqs. (7-9) that the nutation frequency equations are unaffected by the body-fixed torques T_x and T_y.

Consider the case where the BFT is applied as a step function. What will the values of ω_{xn}, ω_{yn}, β_n be immediately following the step? ω_x, ω_y, and β cannot change instantaneously, and since ω_{xw}, ω_{yw}, and β_w are defined and constant following the step, we know that

$$\omega_{xn}(0^+)=\omega_x(0)-\omega_{xw}(0^+)$$

$$\omega_{yn}(0^+)=\omega_y(0)-\omega_{yw}(0^+)$$

$$\beta_n(0^+)=\beta(0)-\beta_w(0^+)$$

If we start with zero initial conditions, the initial nutation components will start with the same magnitude as the wobble components, but the opposite sign. The nutation portion is then damped out.

D. Translational Acceleration Loading (TAL) Response

Consider a translational acceleration of the spacecraft in the axial direction ($\dot{V}_z$). The acceleration of each particle on the appendage in the z direction will exert a torque about the hinge line such that

$$T_\beta=-m_2R_{2y}\dot{V}_z \tag{47}$$

Evaluating Eq. (6) for $T_x=T_y=0$ gives

$$\omega_{xw}=0$$

$$\omega_{yw}=\frac{-J\Omega T_\beta}{[(C-B)(K+J\Omega^2)-J^2\Omega^2]}$$

$$\beta_w=\frac{(C-B)T_\beta}{[(C-B)(K+J\Omega^2)-J^2\Omega^2]} \tag{48}$$

The resulting wobble is described as Translational Acceleration Loading (TAL) wobble, and is found only in the case of nonsymmetrically distributed sources of flexibility. If there were an identical appendage extending in the $-Y$ direction also, we would find $\omega_{yw}=0$.

VI. Design Specifications

With the understanding developed above of the dynamics of a spacecraft plus large damped appendage, the specifications for a nutation damping system may be developed for a specific spacecraft with a specific mission.

The key design criteria are stability margin and time constant. A graph (such as Fig. 2) is useful in understanding the tradeoff that must be made. For each spacecraft configuration and spin rate, a plot of τ vs A_{wII} may be drawn with nutation damper spring constant as the parameter. Requirements on the maximum acceptable values of A_{wII} and τ will set minimum and maximum limits, respectively, on the choice of K. It may happen that a single choice of K meets both requirements for all mission phases. For Galileo, this is not possible. Galileo[4] is a dual-spin spacecraft that will launch in 1985. Its mission includes probe release and insertion into Jupiter orbit. The propellant required for these maneuvers comprises over 50% of the spacecraft mass at beginning of life. Inertia properties change signficantly as the propellant is expended and the probe is jettisoned. Although the majority of the mission takes place at a low spin rate, the large propulsive maneuvers require the pointing stability of a moderately high spin rate. Any spring sufficiently stiff to reasonably limit the wobble amplification during Jupiter orbit insertion renders the nutation damper almost useless for nutation damping during the low spin-rate portions of the mission. To solve this problem, a nutation damper lockout device was included for use during Jupiter orbit insertion and other mission-critical events. The nutation damper effectively has a two-position commandable spring rate, and there are four parameters to specify: stiction torque, T_{ST}; low-spring constant, K_1; high-spring constant, K_2; and damping constant, b.

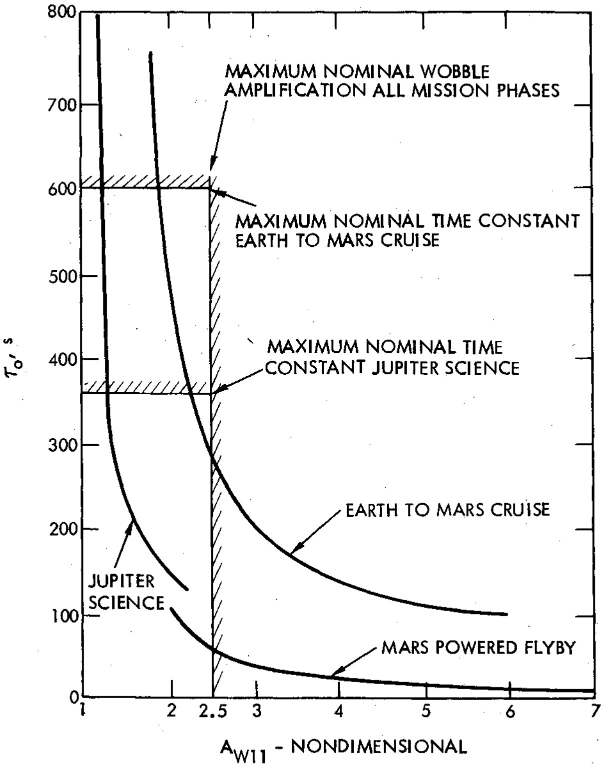

Fig. 2 Time constant-wobble amplification tradeoff.

A. Stiction

Since Eq. (44) relates residual nutation to stiction torque as a function of inertia properties and spin rate only, the stiction torque may be specified by:

$$T_{ST}<(gC\Omega/B)\theta_{NR\text{-req}}$$

B. Spring Constant

The high and low spring constants are chosen using the same procedure. First, the minimum and maximum spring constants are determined, based on time constants and wobble requirements for each combination of inertia properties, and spin rate, which will exist during the mission. Using these extrema, the mission is divided into two portions: that which will use the high-spring constant and that which will use the low.

The BFT and TAL motion during propulsive maneuvers must be investigated to determine if the resulting dynamics are acceptable. During axial ΔV's, these motions occur together.

Both may be reduced if necessary, by increasing K. In addition, BFT motion may be reduced by tightening c.m. alignment requirements.

The despin motor torque margin must be determined to avoid any possible trap states. Increasing K will decrease β, and thus decrease the rotor dynamic imbalance.

Using the above considerations (particularly trap states in the presence of BFT and TAL motion), the high spring constant for Galileo was chosen significantly above the minimum defined by wobble amplification. The low spring constant was chosen only slightly above the minimum.

C. Damping Constant

The optimum damping constant is chosen as described in Sec. III. For Galileo, the time constant is optimized for the Jupiter science mission phase, which uses the low spring at the low spin rate.

VII. Summary and Conclusions

With this paper, the study of passive nutation dampers is extended to include nutation dampers that make use of large existing appendages. The time constant and stability of spacecraft including such dampers are analyzed, along with a number of other practical considerations that may dictate nutation damper design specifications. Finally, the manner in which these considerations affect the design is described with reference to the Galileo spacecraft, where the results of this study are used to write design specifications for a passive nutation damper.

The key conclusion of this work is that the use of large appendages for nutation damping can significantly degrade spacecraft stability margin, and that the final design of the nutation damper must represent a tradeoff between damping performance and stability margin for sufficiently large appendages.

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